

# Kalman Filter Based Estimation of Flow States in Open Channels Using Lagrangian Sensing

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**Abstract**—In this article, we investigate real-time estimation of flow states, average velocity and stage (water depth), in open channels using the measurements obtained from Lagrangian sensors (drifters). One-dimensional Shallow Water Equations (SWE), also known as Saint-Venant equations, are used as the mathematical model for the flow. After linearizing and discretizing the PDEs using an explicit linear scheme, we construct a linear state-space model of the flow. The Kalman filter is then used to estimate the states by incorporating the measurements obtained from passive drifters. Drifters which are equipped with GPS receivers move with the flow and report their position at every time step. The position of the drifters at every time step are used to approximate the average velocity of the flow at the corresponding locations and time step. The method is implemented in simulation on a section of the Sacramento river in California using real data and the results are validated with a two-dimensional simulation of the river. Finally, the performance of the method using Lagrangian sensors is compared to the case of using Eulerian sensors.

## I. INTRODUCTION

Data assimilation is a method for estimating the states of physical systems which originated in meteorology, oceanography and hydrology [1], [3], [6], [19]. Data assimilation incorporates measurements and observations from a physical process into a mathematical model in order to estimate its states. The aforementioned physical systems are examples of distributed parameters systems where the dynamics of the physical system is modeled by a set of partial differential equations and the required boundary conditions. In the case of hydrodynamics, the boundary conditions are typically measured by sensors installed at the corresponding boundary of the domain. Nevertheless, there are always uncertainties and inaccuracies involved in the mathematical model and the measured (or approximated) boundary conditions. Measurements obtained from the physical system are usually incorporated into the model via different techniques to compensate for the inaccuracies. Among these techniques, some common methods include variational data assimilation [7], filtering based methods [17], [20], optimal statistical interpolation [11], or the Newtonian relaxation [26], [13].

In [9], the authors have used the Ensemble Kalman filter to perform real-time estimation of a fully nonlinear two dimensional model of shallow water flows using Lagrangian

measurements of the flow. *Lagrangian measurements* are measurements of the flow properties at a point moving with the flow along the streamline whereas *Eulerian measurements* are measurements of the flow properties at a fixed location. Lagrangian sensors which move with the flow and report their location and possibly other local quantities of interest (temperature, salinity, etc) are commonly used in oceanography [8], [23], [18] (usually referred to as drifters) and in river hydraulics [21].

In this article, we investigate the application of the Kalman filter using a one dimensional model of the flow. Since the application of the Kalman filter is pursued, the shallow water equations are linearized around the steady flow (the *Backwater Curve*). After using an explicit linear scheme to discretize the equations, a linear state-space model is constructed whose states are the collection of the flow states and the input is the set of boundary conditions. Note that although an explicit scheme imposes a limitation on the choice of the temporal and spatial step sizes, the use of an explicit scheme (as opposed to an implicit scheme) is necessary for the construction of a state-space model.

In spite of the fact that the system is essentially nonlinear, a linear model is expected to perform fairly well for a reasonable period of time. This is mainly because, in open channel systems, the variations in the system's excitation (the boundary conditions) are expected to occur somewhat slowly.

As the measurement model, Lagrangian measurements obtained from a desired number of drifters are used. The flow measurements are incorporated into the one-dimensional shallow water model with poorly known boundary conditions via the Kalman filter to estimate the flow states throughout the whole domain. Using drifters instead of static sensors is quite advantageous. Drifters provide cost effective solutions to sensing problems, they are not constrained to be used only on a specific channel, and they offer a competitive accuracy in their measurements.

The rest of this article is organized as follows. In section II, we describe the mathematical model of flow in open channels and construct a linear state-space model after linearizing and discretizing the governing equations. Section III is devoted

to the building of a set-up for the Kalman filtering using the measurements obtained from the drifters. In section IV, we describe the experiment carried out on a two-dimensional simulation of a section of the Sacramento River with real data to evaluate the method and results are presented. Finally, in section V, the conclusion and future subjects of research are stated.

## II. MATHEMATICAL MODEL

### A. The Saint-Venant Model

The Saint-Venant equations, which are first order hyperbolic nonlinear PDEs, are commonly used to model the flow in open-channel hydraulic systems [16],[2]. These equations are obtained from the conservation of mass and momentum. For one-dimensional flow in a channel of rectangular cross-section, these equations are:

$$T \frac{\partial H}{\partial t} + \frac{\partial (THV)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial H}{\partial x} + g(S_f - S_b) = 0 \quad (2)$$

for  $(x, t) \in (0, L) \times \mathbb{R}^+$ , where  $L$  the river reach ( $m$ ),  $V(x, t)$  the average velocity ( $m/s$ ) across cross section  $A(x, t) = T(x)H(x, t)$ ,  $H(x, t)$ ,  $H(x, t)$  the stage or water-depth ( $m$ ),  $T(x)$  the free surface width ( $m$ ),  $S_f(x, t)$  the friction slope ( $m/m$ ),  $S_b$  the bed slope  $m/m$ ,  $g$  the gravitational acceleration ( $m/s^2$ ). Also,  $Q(x, t) = V(x, t)A(x, t)$  is the discharge across cross-section  $A(x, t)$ . The friction slope is empirically modeled by the Manning-Strickler's formula

$$S_f = \frac{m^2 V |V| (T + 2H)^{4/3}}{(TH)^{4/3}} \quad (3)$$

with  $m$  the Manning's roughness coefficient ( $sm^{-1/3}$ ).

*Remark 1:* Although, the use of shallow water equations with discharge and stage as states is more common in hydrodynamics, we use the equivalent version of the equations with the average velocity and stage as the states. This choice is made to reduce the processing of the measurements obtained from the drifters.

*Remark 2:* In the channels where the channel width is considerably greater than the water depth,  $T \gg H$ , an approximation of  $S_f$ , as follows, can be used which will simplify the linearization

$$S_f = \frac{m^2 V |V|}{(H)^{4/3}} \quad (4)$$

*Remark 3:* Throughout this article, the dependence on the spatial variable  $x$  is occasionally omitted for the sake of readability.

### B. Steady flow: Backwater curve

*Backwater curve* is the longitudinal profile of the surface of the water in a non-uniform flow in an open channel when the water surface is naturally or artificially raised above its normal level. Denoting the variables corresponding to the

steady state by adding a bar,  $\bar{\cdot}$ , the steady state equations can be written as:

$$\frac{d\bar{V}(x)}{dx} = -\frac{\bar{V}(x)}{\bar{H}(x)} \frac{d\bar{H}(x)}{dx} - \frac{\bar{V}(x)}{T(x)} \frac{dT(x)}{dx} \quad (5)$$

$$\frac{d\bar{H}(x)}{dx} = \frac{S_b - \bar{S}_f}{1 - \bar{F}(x)^2} \quad (6)$$

with  $\bar{C} = \sqrt{g\bar{H}}$  the wave celerity,  $\bar{F} = \bar{V}/\bar{C}$  the Froude number. Throughout this article, we assume the flow to be *sub-critical*, i.e.,  $\bar{F} < 1$ .

*Remark 4:* In the case of uniform flow, the steady velocity,  $\bar{V}(x) = \bar{V}$ , and the normal depth,  $\bar{H}(x) = H_n$ , can be calculated by solving the normal depth equation,  $\bar{S}_f = S_b$ .

### C. Linearized Saint-Venant Model

The Saint-Venant equations are nonlinear in the flow variables  $V$  and  $H$ . It is a common practice to linearize the equations when a linear model of the system is desired [4], [5]. Each term  $f(V, H)$  in the Saint-Venant model can be expanded in Taylor series around the steady state flow variables  $\bar{V}(x)$  and  $\bar{H}(x)$ . Considering only the first order perturbations,  $f(V, H) \approx f(\bar{V}, \bar{H}) + (f_V)|_{(\bar{V}, \bar{H})}v(x, t) + (f_H)|_{(\bar{V}, \bar{H})}h(x, t)$ . The first order perturbations in velocity (resp. stage) is given by  $v(x, t) = V(x, t) - \bar{V}(x)$  (resp.  $h(x, t) = H(x, t) - \bar{H}(x)$ ).

After substituting the expressions of  $H$  and  $V$  with  $\bar{H} + h$  and  $\bar{V} + v$  in equations (1) and (2) and some manipulation of terms, the linearized Saint-Venant model for the perturbed flow variables  $v$  and  $h$  can be written in the following form

$$h_t + \bar{H}(x)v_x + \bar{V}(x)h_x + \alpha(x)v + \beta(x)h = 0 \quad (7)$$

$$v_t + \bar{V}(x)v_x + gh_x + \gamma(x)v + \eta(x)h = 0 \quad (8)$$

with  $\alpha(x)$ ,  $\beta(x)$ ,  $\gamma(x)$  and  $\eta(x)$  given by

$$\alpha(x) = \frac{d\bar{H}}{dx} + \frac{\bar{H}}{T} \frac{dT}{dx} \quad (9)$$

$$\beta(x) = -\frac{\bar{V}}{\bar{H}} \frac{d\bar{H}}{dx} - \frac{\bar{V}(x)}{T(x)} \frac{dT(x)}{dx} \quad (10)$$

$$\gamma(x) = 2gm^2 \frac{|\bar{V}|}{\bar{H}^{4/3}} - \frac{\bar{V}}{\bar{H}} \frac{d\bar{H}}{dx} - \frac{\bar{V}(x)}{T(x)} \frac{dT(x)}{dx} \quad (11)$$

$$\eta(x) = -\frac{4}{3}gm^2 \frac{\bar{V}|\bar{V}|}{\bar{H}^{7/3}} \quad (12)$$

### D. Discretization: Lax Diffusive Scheme

We use the Lax diffusive scheme [15] [24] which is a first-order explicit scheme to discretize the equations. Using  $f$  to represent the dependent variables,  $v$  and  $h$ , the derivatives become

$$\frac{\partial f}{\partial t} = \frac{f_i^{k+1} - \frac{1}{2}(f_{i+1}^k + f_{i-1}^k)}{\Delta t} \quad (13)$$

$$\frac{\partial f}{\partial x} = \frac{(f_{i+1}^k - f_{i-1}^k)}{2\Delta x} \quad (14)$$

Applying this scheme to equations (7) and (8), we get:

$$\begin{aligned}
h_i^{k+1} = & \frac{1}{2}(h_{i+1}^k + h_{i-1}^k) \\
& - \frac{\Delta t}{4\Delta x}(\bar{H}_{i+1} + \bar{H}_{i-1})(v_{i+1}^k - v_{i-1}^k) \\
& - \frac{\Delta t}{4\Delta x}(\bar{V}_{i+1} + \bar{V}_{i-1})(h_{i+1}^k - h_{i-1}^k) \\
& - \frac{\Delta t}{2}(\alpha_{i+1}v_{i+1}^k + \alpha_{i-1}v_{i-1}^k) \\
& - \frac{\Delta t}{2}(\beta_{i+1}h_{i+1}^k + \beta_{i-1}h_{i-1}^k) \quad (15)
\end{aligned}$$

$$\begin{aligned}
v_i^{k+1} = & \frac{1}{2}(v_{i+1}^k + v_{i-1}^k) \\
& - \frac{\Delta t}{4\Delta x}(\bar{V}_{i+1} + \bar{V}_{i-1})(v_{i+1}^k - v_{i-1}^k) \\
& - \frac{g\Delta t}{2\Delta x}(h_{i+1}^k - h_{i-1}^k) \\
& - \frac{\Delta t}{2}(\gamma_{i+1}v_{i+1}^k + \gamma_{i-1}v_{i-1}^k) \\
& - \frac{\Delta t}{2}(\eta_{i+1}h_{i+1}^k + \eta_{i-1}h_{i-1}^k) \quad (16)
\end{aligned}$$

This scheme is stable provided that the Courant-Friedrich-Lewy (CFL) condition holds, i.e.

$$\frac{\Delta t}{\Delta x}|V| \leq 1 \quad (17)$$

### E. Discrete State-Space Model

Using the discretization of the constitutive equations, we can form a state-space model as follows

$$z(k+1) = Az(k) + Bu(k) \quad (18)$$

where

$$z(k) = (v_2^k, \dots, v_I^k, h_2^k, \dots, h_I^k)^T \quad (19)$$

and  $u_c(k)$  is the boundary conditions,

$$u(k) = (v_1^k, h_1^k)^T \quad (20)$$

$h_i^k$  and  $v_i^k$  are velocity and stage perturbations at cell  $i$  at time  $k\Delta t$ , respectively, and  $I$  is the number of cells used for the discretization of the channel.

## III. STATE ESTIMATION SET-UP

### A. Process Model

Modeling the uncertainties by adding a noise term  $w(k)$  to the state-space equation (18) leads to

$$z(k+1) = Az(k) + Bu(k) + w(k) \quad (21)$$

The process noise is assumed to be white Gaussian noise and

$$E[w_k w_l^T] = Q_k \delta_{kl} \quad (22)$$

$$(23)$$

$z_0 \in \mathbb{R}^m$  is the initial conditions and it is assumed as

$$z_0 = \mathcal{N}(\bar{z}_0, P_0) \quad (24)$$

### B. Measurement Model

The information of the position the drifters equipped with GPS can be used to obtain Lagrangian measurements of the flow velocity. Each drifter reports its current position  $x(k)$  at every time step  $k$  which is used to calculate the speed of the drifter at every time step. Since our estimation method is based on a one-dimensional model of the flow, we have the drifter released at the center line of the channel and we assume it stays on the center line as it moves along the channel. This is a realistic assumption as long as the drifter is moving on the same channel since the lateral components of the flow velocity are usually negligible.

Denoting the collection of average velocities obtained from the drifters at time step  $k$  by  $y(k)$ , the measurement model can be written as

$$y(k) = C(k)z(k) + e(k) \quad (25)$$

where  $e(k)$  is the measurement noise on the sensors which is assumed to be white Gaussian noise and

$$E[e_k e_l^T] = R_k \delta_{kl} \quad (26)$$

We also assume that the process and measurement noises and the initial conditions are all independent.

Note that the observation operator  $C(k)$  is time-varying since the drifters are moving with the flow and therefore the cells at which the flow velocity is measured is changing over time.

Defining the mean and the covariance of the estimations with the following notations

$$\hat{z}_k = E[z_k | y_0, \dots, y_k] \quad (27)$$

$$\hat{z}_k^- = E[z_k | y_0, \dots, y_{k-1}] \quad (28)$$

$$P_k^- = \Sigma_{k|k-1} \quad (29)$$

$$P_k = \Sigma_{k|k} \quad (30)$$

the iterations of the Kalman filter can be written as follows [12]

Time update:

$$\hat{z}_k^- = A\hat{z}_{k-1} + Bu_k \quad (31)$$

$$P_k^- = AP_{k-1}A^T + Q \quad (32)$$

Measurement update:

$$K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R)^{-1} \quad (33)$$

$$\hat{z}_k = \hat{z}_k^- + K_k (y_k - C_k \hat{z}_k^-) \quad (34)$$

$$P_k = (I - K_k C_k) P_k^- \quad (35)$$

## IV. IMPLEMENTATION

### A. Experiment Set-up

The method is implemented on a part of the Sacramento river, upstream of the intersection with the Georgiana slough. The Sacramento river is a part of the Sacramento-San Joaquin Delta in California which is an integral part of California's water system. The bathymetry of the channel, shown in Figure 1, is provided by United States Geological

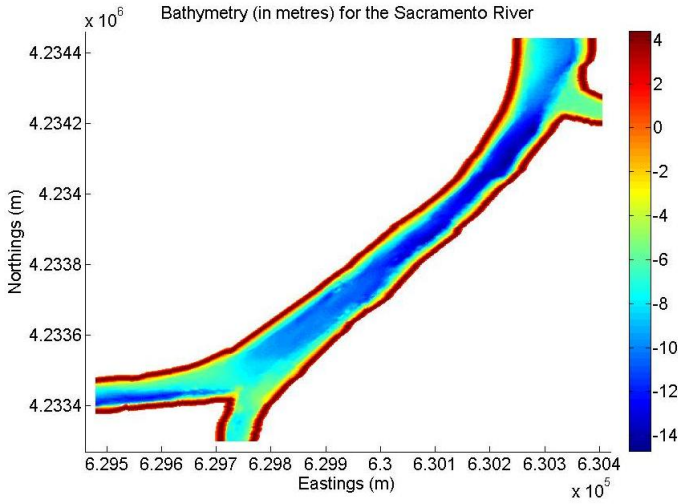


Fig. 1: Bathymetry in Sacramento River. The bathymetry on this section of the Sacramento River varies from -14m in the deepest part to +2 on the river banks.

Survey (USGS). The section of interest is of 900m length and is 85m wide in the narrowest part and 190m wide in the widest part. For discretization, we divide the channel to 30 cells with each cell being 30m long. This results in a state-space model with 58 states as described in section II-E.

The so-called *forward simulation* is performed in a commercial hydrodynamic software TELEMAC 2D [22] to generate the *true state* as well as the drifter position data. TELEMAC uses a streamline upwind Petrov-Galerkin based finite element solver for hydrodynamic equations. The mesh used for the simulation has 1939 nodes and 3525 triangular elements. The boundary conditions, shown in Figure 3, are computed using the Delta Simulation Model II (DSM2) [14]. DSM2 is a model of the San Francisco Bay and Sacramento Delta that provides discharge and surface elevation at various locations every one hour. The flow diagram of the implementation procedure is shown in Figure 2. Note that the true state generated by the 2D simulation in TELEMAC must be converted to 1D data to be compared with the estimation results which are based on a 1D model of the flow.

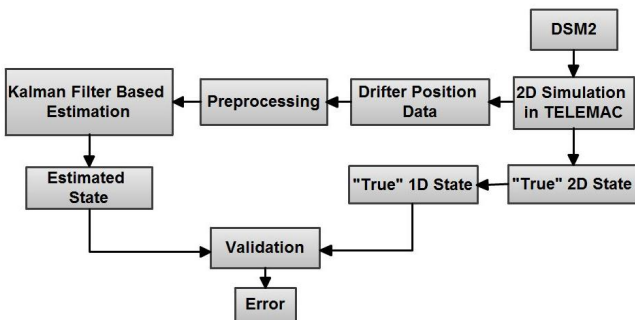


Fig. 2: Flow diagram of the implementation procedure.

The experiment starts at 3:40PM on March 16th 2007. The simulation runs for two and a half hours before the experiment so that the effects of the initial conditions are

washed away and the model is stabilized. The time frame of the experiment is chosen such that the flow variations are as noticeable as possible. Also, as it can be seen in Figure 3, there is an abrupt change in the boundary conditions at 4PM. This enables us to evaluate the performance of the Kalman filter in estimating the flow states in case of sudden changes in the condition of flow.

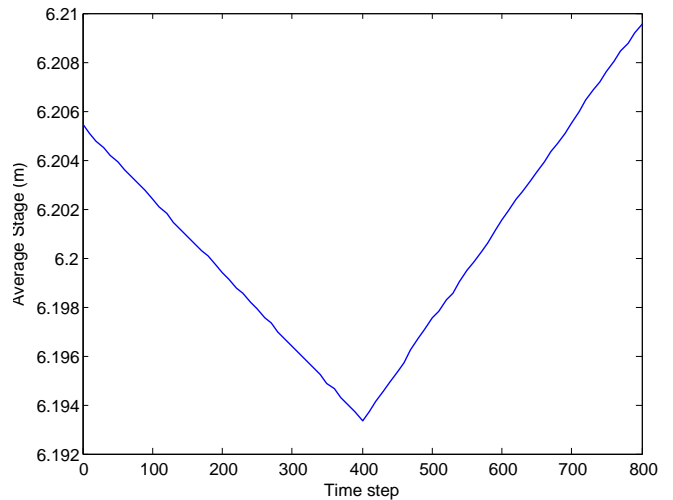
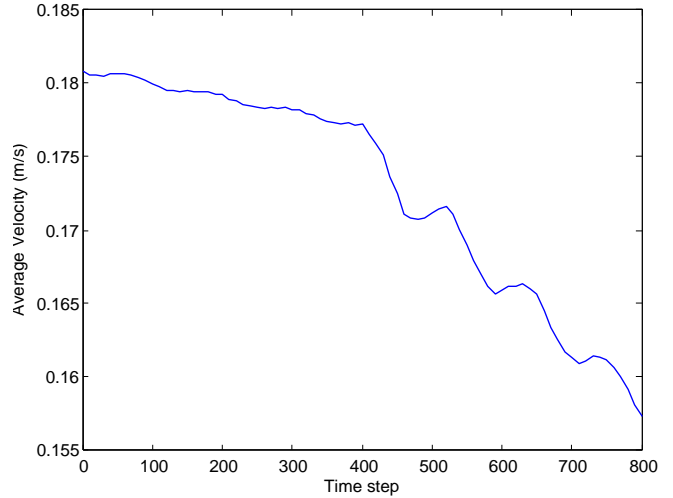


Fig. 3: The boundary conditions (a) the average velocity (b) the stage.

At 3:40PM, a single drifter is released at the upstream end of the river. The drifter moves with the current and its position is recorded at every time step. The trajectory of the drifter is illustrated in Figure 5. The data assimilation starts as soon as the drifter is released and it ends when the drifter reaches the downstream end of the river, at 4:18PM. At each time step, the velocity of the drifter is approximated by the difference between its current and previous position divided by the time step,  $\Delta t$ , which is chosen to be 3 seconds in this experiment. This gives us a measurement of the velocity of the current at the position of the drifter at every time step.

We assume that the initial values of velocity and surface elevation at the last cell are available to us. These values

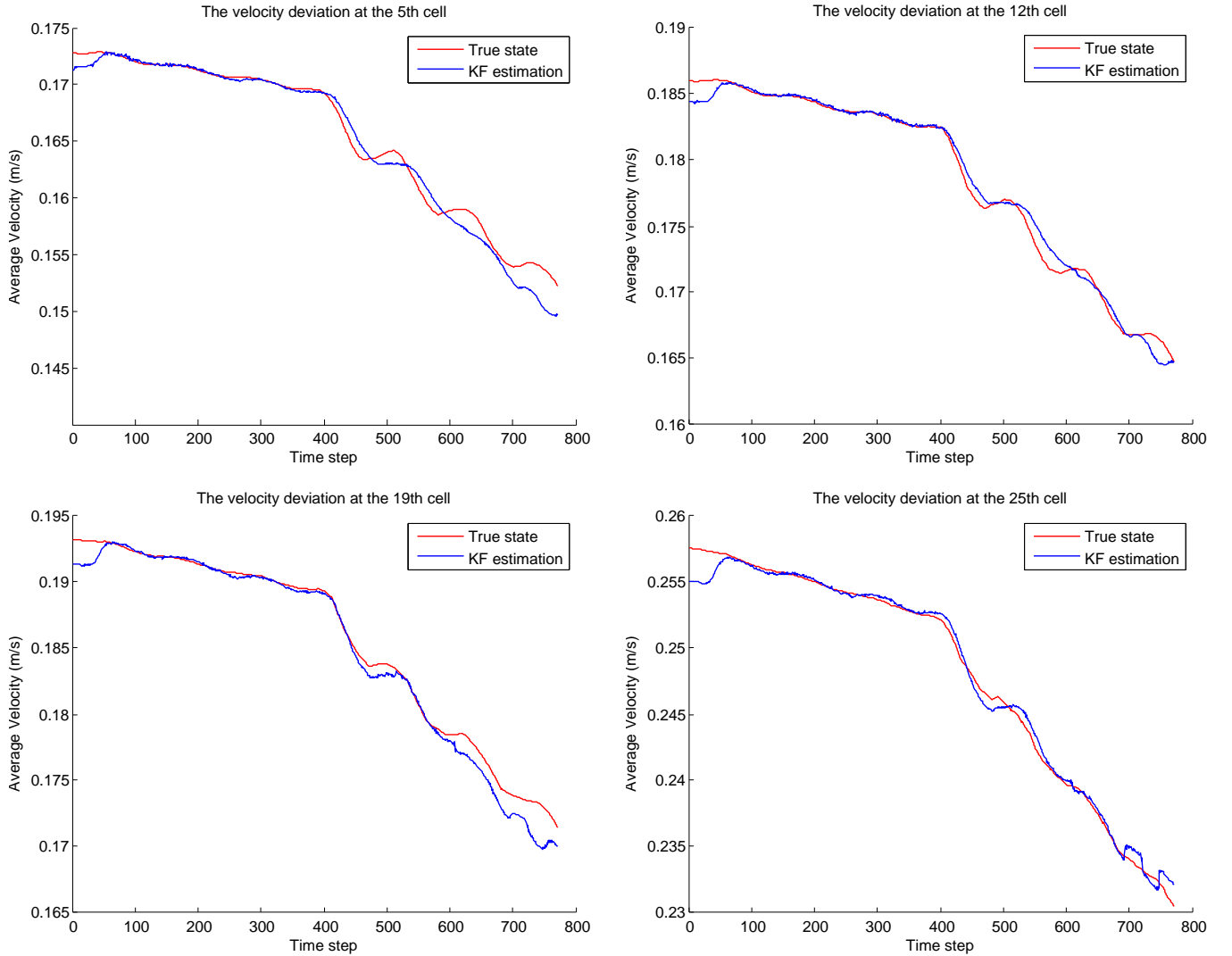


Fig. 4: The time evolution of the estimated velocity and the true velocity at (a) the 5<sup>th</sup> cell, (b) the 12<sup>th</sup> cell, (c) the 19<sup>th</sup> cell and (d) the 25<sup>th</sup> cell.

are required to compute the backwater curves. However, we assume the boundary conditions are not available to us, i.e. there is no static sensor infrastructure available in the system. Therefore, in the stochastic state-space model, equation (21), the input  $u(k)$  is assumed to be a constant, e.g. an approximation of its initial value. This assumption makes the method suitable for being used for estimation in the channels where there is no static sensor infrastructure available since the only measurement device needed for the estimation is a single drifter.

### B. Numerical Results

Figure 4 shows the estimated velocity and the *true velocity* at four different cells.

Figure 6 shows the time evolution of the relative error of the estimated velocity which is calculated using the following formula

$$\text{error}(k) = \sqrt{\frac{\sum_{i=1}^{N_{cell}} (u_i^k - \hat{u}_i^k)^2}{\sum_{i=1}^{N_{cell}} (u_i^k)^2}} \quad (36)$$

where  $u_i^k$  and  $\hat{u}_i^k$  are the true and estimated values of the velocity at cell  $i$  and time step  $k$ .

As it can be seen in Figure 6, the relative error decreases rather quickly and reaches below 2% at time step 50 and remains below 2% until time step 400. After time step 400, ignoring the fluctuations, the error increases relatively rapidly. Note that time step 400 corresponds to 4pm which is the time when an abrupt change in the boundary conditions occurs. In fact, the increase in the error after time step 400 is due to the fact that the deviation of the state of the system from the steady state around which the system has been linearized gets too large as a result of the abrupt change in the boundary conditions. Furthermore, as it can be seen in Figure 4, after time step 400, there are some noticeable fluctuations in the system which is in fact as a result of the fluctuations in the velocity boundary conditions which can be seen in Figure 3. In spite of the abrupt change and also the oscillations in the forcing function of the system, the relative error stays well below 14% until the end of the experiment.

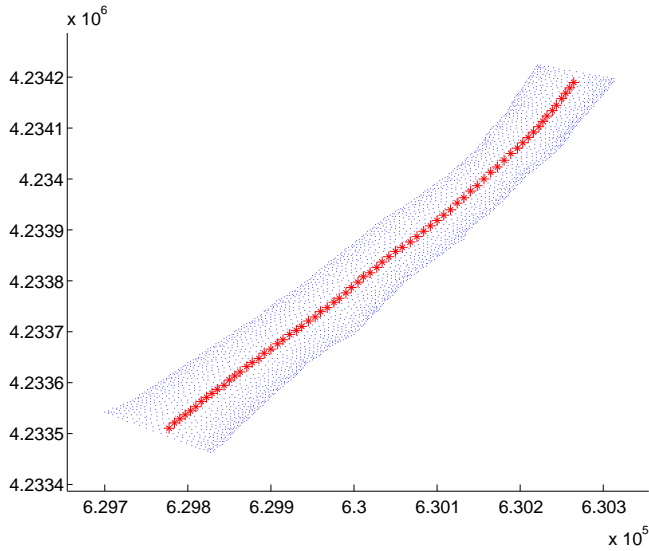


Fig. 5: Trajectory of the drifter.

The computational cost of the method is very reasonable. In the above experiment with a state-space model of 58 states, each iteration of the Kalman filter takes less than 1 millisecond on a 2.4GHz Pentium dual core processor.

Note that although the dynamic of the physical system is nonlinear, the Kalman filter which uses a linear model of the system performs very well. The main reason is that system states stay in a reasonable distance from the steady state around which the system has been linearized. This is because the variations in the excitations, i.e. the boundary conditions, are quite moderate which is the case in most open channel systems.

### C. Lagrangian vs. Eulerian

We performed the method again with exactly the same set-up, but this time by using measurements of the flow obtained from an Eulerian (static) sensor instead of a drifter. In this case, there is only a static sensor located at the 6<sup>th</sup> cell of the channel which measures the average velocity of the water at this location.

Figure 6 shows the relative error of the estimated velocity in case of using measurements obtained from one static sensor. As it can be seen from the figure, the trend of the time evolution as well as the numerical values of the relative error are quite similar to the case of using a single Lagrangian sensor.

This comparison justifies the fact that while production, deployment and maintenance of Lagrangian sensors cost much less than Eulerian sensors, they offer quite similar performance in estimating the flow states in open channels.

## V. CONCLUSIONS AND FUTURE WORKS

In this article, we presented a method to estimate the flow states in an open channel using passive drifters. The drifters are only required to be equipped with a GPS receiver and a transmitter-receiver device for communication with the computational unit. Using the position of drifters at

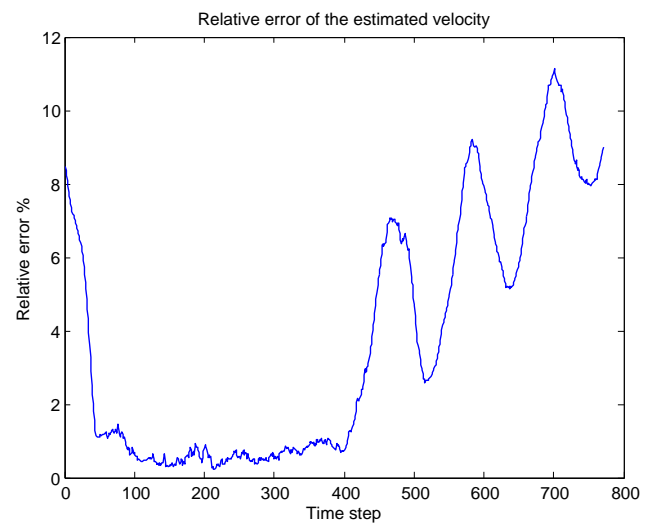
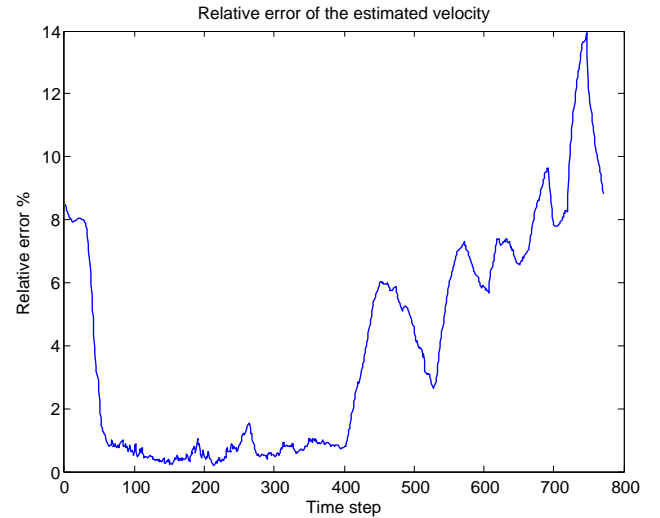


Fig. 6: The time evolution of the relative error of the estimated velocity while using (a) a Lagrangian sensor (drifter) (b) an Eulerian (static) sensor.

every time step, measurements of the flow velocity at the corresponding locations are obtained. The Kalman filter, based on a linear 1D shallow water equations, was used to estimate the current flow states by incorporating the online measurements of the flow velocity. The results of the implementation of the method using a 2D simulation of a section of the Sacramento River with real data was presented. We compared the estimation results while using a single drifter with the case of using a static sensor, which is capable of measuring the flow velocity (or discharge) at a location at which it is installed. The performance of the method in both cases was shown to be very similar.

Requiring basic and cheap equipments, having a very low computational complexity and yet offering an outstanding performance, the method presented here seems quite practical for real-time studies of the flow in open channels and irrigation systems and also for control purposes. Particularly, in channel systems where no sensing infrastructure is available, a number drifters can be deployed to carry out the method.

The drifters can be released in the water at the beginning of the study and be retrieved in the end to be used in the future again.

Data assimilation in a network of interconnected channels is considered in a companion paper, [10], where a Quadratic Programming (QP) based method is presented to estimate the open boundary conditions of the network. The QP-based method is appropriate for the situations where an accurate model of the flow is desired for a longer period of time, after all data have been collected. Unlike the Kalman filter based method, the QP-based method does not provide estimates in real time, however, it can use an implicit linear model of the flow which allows choosing a relatively large time step. Implementing the Kalman filter based method on complex networks of interconnected channels by deploying a fleet of drifters is a subject of future research. Performing the method in a field test, [25], is also a future research topic.

## VI. ACKNOWLEDGMENTS

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